



Technological University Dublin  
**ARROW@TU Dublin**

---

Conference papers

School of Mathematics

---

2017

## A Project-Based-Learning Approach to Teaching Second-Order Differential Equations to Engineers

Michael Carr

*Technological University Dublin, michael.carr@tudublin.ie*

Eabhnat Ní Fhloinn

*Dublin City University*

Follow this and additional works at: <https://arrow.tudublin.ie/scschmatcon>

 Part of the [Engineering Commons](#)

---

### Recommended Citation

Carr, M. & Ní Fhloinn, E. (2017). A Project-Based-Learning Approach to Teaching Second-Order Differential Equations to Engineers. *Proceedings of 45th SEFI Annual Conference*, Azores, Portugal, 18-21, September.

This Conference Paper is brought to you for free and open access by the School of Mathematics at ARROW@TU Dublin. It has been accepted for inclusion in Conference papers by an authorized administrator of ARROW@TU Dublin. For more information, please contact [yvonne.desmond@tudublin.ie](mailto:yvonne.desmond@tudublin.ie), [arrow.admin@tudublin.ie](mailto:arrow.admin@tudublin.ie), [brian.widdis@tudublin.ie](mailto:brian.widdis@tudublin.ie).



This work is licensed under a [Creative Commons Attribution-Noncommercial-Share Alike 3.0 License](#)



# A project-based-learning approach to teaching second-order differential equations to engineers

Michael Carr<sup>1</sup> and Eabhnat Ní Fhloinn<sup>2</sup>

<sup>1</sup>*School of Multidisciplinary Technologies, Dublin Institute of Technology, Ireland*

<sup>2</sup>*School of Mathematical Sciences, Dublin City University, Ireland*

## Abstract

In an attempt to increase engagement in a third-year mechanical-engineering mathematics module, a series of group-work sessions were introduced on a single topic. Students were asked to use the solution of a second-order differential equation (previously introduced in lectures) to solve a problem in which they must design a simple spring-damper system for one of a lorry, digger, truck, tractor, car, motorbike or pogo stick. This project was worth 10% of the module and done in groups of four. The groups gave an oral presentation of their results and a written report was handed up for grading. A detailed description of the implementation of this assessment is given along with an analysis of how student performance in questions on similar topics in the terminal examination for this module.

## Introduction

Mathematics as a discipline presents challenges for a large number of students, not least in terms of engagement and relevance (Hidi and Harackiewicz 2000). Since September 2012, incoming first-year students to higher education in Ireland have studied a revised mathematics curriculum (Project Maths) in second-level (Jeffes et al 2013). This new approach to the teaching and learning of mathematics in Ireland aims to situate mathematics in everyday contexts where possible, so that students will be better able to understand the uses and relevance of mathematics. However, these students entered into a higher education system that was accustomed to a mathematics curriculum that had not changed substantially in almost fifty years prior to this. Much of the material taught in the early years of mathematics is not explicitly mapped at that point to modules or applications in later years, making it difficult for students to understand the importance of what they are learning at this early stage in their careers. It is commonly the case that, in later years, lecturers will refer back to material covered in first year and show students how they will now use this mathematics in more advanced applications. However, from the point-of-view of student engagement and retention, this seems to be done at too late a stage, and needs to be dealt with in early years instead. This is particularly important in engineering as it relies heavily on mathematics throughout the degree programme. Successful service-teaching of mathematics relies heavily on a “sufficient supply of discipline related problems” (Yates 2003). This changing mathematical landscape in Ireland provided the motivation for the development of the project-based-learning approach described in this paper.

Engineering mathematics is generally either taught by engineering lecturers (who are usually not experts in mathematics) or by lecturers from mathematics departments who are not embedded within the students’ home departments, and naturally may not be experts in the overall discipline being studied by the students. As a result, the balance between theory and practical applications is often skewed (Sazhin 1998, p.145). Lecturers may forget the significance of students “getting a feeling for the importance of the subject” (Rota). The level of interaction between mathematics lecturers and staff members in the students’ home departments can vary widely from institution to institution.

In Dublin Institute of Technology, students are offered two main routes to obtain a Level 8 engineering qualification: via direct entry onto a four-year Honours degree programme (Level 8) or alternatively through a three-year Ordinary degree programme (Level 7) followed by a transfer into third year of the Honours degree (Llorens et al. 2014, Carr et al. 2013). A recent study of the First Year Experience (FYE) in the eight third level institutions in the Dublin Regional Higher Education Area (DRHEA) found that one of the key problem areas identified by academics across all eight institutions was lack of “student engagement” (Roper et al 2013, Cusack et al 2013). This lack of engagement often results in poor performance and ultimately impacts upon retention. This project is thus an attempt to evolve the teaching of engineering mathematics at Level 7 to both improve the engagement of students in engineering mathematics classes and to provide a deeper understanding of the material, which may ultimately help these students to progress onto a Level 8 degree programme if they so desire.

## **Background**

There exist many examples in the literature on the need to make the mathematics we teach to engineers more applied. The development of such examples and projects can be challenging for those teaching engineering mathematics as they may not be engineers or familiar with all aspects of engineering. However, there is a full spectrum of initiatives available in the literature, from improved examples in the classroom (e.g. Helm, Young et al 2012, Robinson 2008) to teaching the material via problem-based learning (e.g. Rooch et al. 2012). We now provide a brief overview of some of these approaches.

Helping Engineers Learn Mathematics (HELM) is a major curriculum development project undertaken by a consortium of five English universities - Loughborough, Hull, Reading, Sunderland and Manchester - led by Loughborough. This project provided a huge list of engineering mathematics resources including a set of good examples of engineering applications of mathematics (<http://helm.lboro.ac.uk/>). Although this work is of a high standard, many of the examples contained therein are more relevant to later years of a Level 8 engineering programme, and the examples suitable for Level 7 students are limited in number.

Young et al (2012) at the University of Central Florida developed a bolt-on single-credit module called “applications of calculus”, taught in parallel with their calculus modules. This has been shown to be effective in terms of retention of students within STEM subjects, although there are no projects introduced, simply a range of problems completed in class that are relevant to applications.

Robinson (University of Loughborough) used sports-based group projects for undergraduate students in sports science (Robinson 2012). These projects consisted of teamwork, use of software and application of mathematics to realistic problems. This not only improved engagement, it also introduced a range of important skills for engineers, such as technological and communication skills as well as collaborative and analytical techniques.

Rooch et al. (2012) developed a series of projects (ribbed cooler and a Segway) for teaching mathematics to first year engineering students in Germany. However, the mathematics required is quite involved in each case, meaning that they must supply the students with a number of different formulae in order to allow them to complete the projects. This is a common difficulty when designing “real-life” mathematics projects for students to attempt, due to the scaffolded nature of mathematical knowledge.

Within Dublin Institute of Technology itself, some example of project-based learning already exist. For example, design projects were introduced into first-year physics lab sessions for engineering students. These projects relied upon material covered in mathematics, physics and mechanics modules, bringing them all together in a single design project (Sheridan et al. 2010). A range of other variations exist between teaching applications and full problem-based-learning from Verner et al. (2008) and Mills and Treagust (2004), through to Abramovich and Grinshpan (2008).

## **Aims and scope of project**

Within Dublin Institute of Technology, we wish to move towards a more student-centred-learning approach for the teaching of mathematics across all three years at Level 7. To do so, it has been decided to first pilot this technique in the third year of the programme and then work backwards to first year, once initiatives have proven to be successful. Much work has been done on using project-based/application-based learning as a method for teaching mathematics in higher education, but in the main, these modules have many mathematical pre-requisites, so they are in essence only suitable for later years of a programme and/or re essentially being “bolted on” (Young et. al 2011) to pre-existing modules. Similar work has been done in the third year of the Level 8 degree programme to teach mathematical modelling with good success (Keane, Carr and Carroll, 2008), so it was of interest to introduce it at a similar stage in a Level 7 programme and monitor its impact. Given that the standard of first year in a Level 7 programme is not high enough for many of these existing resources to be used, by trialling this approach in third-year, we can learn valuable lessons before considering earlier years.

The aim of this work is to use a hybrid approach to “project-based-learning” where a significant amount of the pre-requisites is taught over several weeks in a more standard approach and then a realistic project is introduced that consolidates the material that has been covered in class and provides an opportunity to learn applications of the material.

The objectives of the project were to improve engagement and ultimately retention of students; to give students a deeper understanding of the material; to introduce problem-solving, teamwork and communication skills; to move towards a more student-centred environment within the existing structure of lectures and tutorials; and to create a series of resources that could be used by lecturers teaching at Level 7.

## **Project overview**

A series of two-hour group-work sessions were introduced, focused on the topic of second-order differential equations. Following a number of standard lectures, students were assigned to groups of four to work on a short project together during the group-work sessions, with additional work to be completed outside of class time. The project asked students to use the solution of a second order differential equation (previously introduced in lectures) to design a simple spring-damper system for a vehicle from the following list: lorry, digger, truck (large), truck (small), motorbike, motorbike (scrambler), bus (large), bus (small), moped, quad bike, tractor, tractor (seat), car (large), car (small), pogo stick, racing bike or standard bicycle. No two groups were assigned the same vehicle, and the different masses, number of wheels involved and type of damping required meant that the projects were sufficiently different that each group had to work independently on their solution.

The project was worth 10% of the students’ final grades for that module, with the marks awarded per group. At the end of three weeks, each group presented their solution to the class

during a ten-minute presentation slot, as well as handing up a short (four-ten page) report. The variation in report-length was chosen to allow students to include detailed diagrams and additional information where needed. The mixture of assessment methods included within the project gave students the opportunity to display their skills in a range of areas, while providing them with useful practice of presenting technical data in a clear and coherent manner. Students were obliged to attend all the presentations given, which also provided a valuable opportunity for peer-learning, as they heard how different groups had approached a similar problem, and allowed for some class discussion about optimum approaches afterwards.

In order to design a simple spring-damper system, students needed to first consider the mass of the vehicle, calculate an appropriate spring constant, and decide on what type of damping would be ideal for the vehicle in question. For example, the damping needed by a scrambler motorbike is different from that of a family car, where a smoother ride would be required. This was a multi-layered problem, which allowed students to investigate a number of areas in greater depth, considering aspects relevant to the generation of the second-order differential equation. Once they had solved the differential equation, they were then required to sketch the analytic solution by hand, to investigate if the resultant sketch resembled the type of damping they hoped to produce. If so, they then needed to plot a graph of the analytic solution and relate these back to the original problem, giving an interpretation of their results.

### Analysis of exam paper questions

Judging the success of a project-based intervention such as this is difficult, although student engagement with the project was high and their reaction was universally positive. However, in previous years, examination questions based on realistic uses of second-order differential equations were extremely poorly answered, or, in many cases, not even attempted, despite similar questions having been addressed in lectures. There was no choice given in the examination and so these questions were compulsory but they were still avoided by students. Therefore, an analysis was done of a similar question from the terminal examination paper sat by this year's students, all of whom had the benefit of having completed their short project on this area. A total of 35 examination papers were analysed, with two separate parts of one question considered. The question asked:

A spring dashpot system has a damping force resulting from the dash-pot  $F_d = -kV$  and the restoring force from the spring  $F_s = -nx$ . This system is represented by the differential equation below

$$\frac{md^2x}{dt^2} + \frac{kdx}{dt} + nx = 0$$

- (a) Find the general solution to this equation if  $k^2 = 4mn$  and thus sketch your solution.
- (b) Explain your sketch and how it relates to what is happening in the damping system. You should illustrate what is happening by using an engineering example of where this may be used.

For part (a), 11 students (31.5%) answered correctly, showing they were able to derive an analytical solution and sketch the required graph. A further 11 students (31.5%) made a reasonable attempt but derived the wrong analytical solution and drew the wrong graph (even for the solution they derived). 5 students (14%) derived the wrong solution but knew what the correct graph should look like and drew this. The final 8 students (23%) were unable to make a proper attempt at this question.

For the descriptive answer to part (b), 11 students (31.5%) gave an entirely correct answer and 5 students (14%) made a reasonable attempt at an explanation. However, 19 students (54.5%) gave a poor explanation or did not attempt to give any.

When each student's project mark was plotted against their performance on this exam question, the result was statistically significant, with a Pearson correlation of 0.453 found with  $p=0.006$ . Similarly, when each student's overall performance was plotted against their performance on this exam question, another statistically significant correlation was found, with a Pearson correlation of 0.71 with  $p=0.000$ . While this is not surprising, showing that the most capable students performed well on all components of the assessment, it does contrast with previous years, when even strong students avoided or did poorly on the differential equations question in the examination.

## Conclusions and future work

The introduction of a short project-based-learning element into a mathematics module for third-year Level 7 mechanical engineering students was well-received by students and resulted in greater engagement with examination questions on the same topic in comparison with previous years when this material was taught in a standard lecture environment. Student performance on these examination questions was also improved, though the strongest students overall performed the best on these questions. Through the inclusion of a presentation element within the project assessment, the "active verbal involvement" of students advocated by Kwon (2000) was addressed, allowing students the opportunity to explain and justify their thinking, which is particularly important for engineering students while studying mathematics. Although it is challenging to develop projects of this type, the aim was to create a scenario that would be "experientially real to students and...take into account students' current mathematical ways of knowing" (Rasmussen & Kwon 2007). As a first foray into project-based-learning for mathematics for third-year Level 7 engineering students, it was a success and now the focus will shift to attempting to do similar in earlier years of the programme, where it is hoped that it will be equally well-received by students and provide a positive contribution to the teaching and learning of mathematics for Level 7 engineering students.

## References

- Abramovich, S. and Grinshpan, A. Z. (2008). Teaching mathematics to non-mathematics majors through applications, *PRIMUS* 18(5): 411–428.
- Carr, M., Ní Fhloinn, E., Murphy, E. and Bowe, B. (2013) Addressing continuing mathematical deficiencies with advanced mathematical diagnostic testing. *Teaching Mathematics and Its Applications* 32 (2): 66-75.
- Hidi, S. and Harackiewicz, J.M. (2000) Motivating the academically unmotivated: a critical issue for the 21<sup>st</sup> century. *Review of Educational Research*. 70(2): 151-179.
- Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheeler, R. and Dawson, A. (2013). Research into the impact of Project Maths on student achievement, learning and motivation: final report. Slough: NFER.
- Keane G., Carr, M. and Carroll, P. (2008) "An integrated approach to the teaching of numerical methods to engineering students", *Proceedings of INTED*, Valencia, Spain.

- Kwon, Oh Nam (2000) “An Inquiry-Oriented Approach to Undergraduate Mathematics: Contributions to Instructional Design of Differential Equations” *Enhancing University Mathematics: Proceedings of the First KAIST International symposium on education*. 91.
- Llorens, M., Nevin, E. and Mageean, E. (2014). “Online Resource Platform for Mathematics Education”. In *Proceedings 44th Annual Frontiers in Education (FIE) Conference*, 1865-1872. Madrid, Spain
- Mills, J.E. and Treagust, D.F. (2003) Engineering education—Is problem-based or project-based learning the answer? *Aust. J. Eng. Educ.* 3: 2–16.
- Rasmussen, C. (2001). New directions in differential equations: A framework for interpreting students’ understandings and difficulties. *Journal of Mathematical Behavior*, 20: 55-87
- Rasmussen, C. and Kwon, O.N (2007) An inquiry-oriented approach to undergraduate mathematics *Journal of Mathematical Behavior* 26: 189–194
- Robinson, C. (2012) Using sport to engage and motivate students to learn mathematics. *Teaching Mathematics and its applications* 31: 49-56.
- Rooch, A et al. (2012) MathePraxis-Connecting first year mathematics with engineering applications, *European Journal of Engineering Education*, 37(3): 255-266
- Rota, G.C. Ten lessons I wished I had learned before teaching differential equations. <https://web.williams.edu/Mathematics/lg5/Rota.pdf> accessed May 2016
- Sazhin, S.S. (1998). Teaching mathematics to engineering students. *Int. J. Engng Ed.* 14(2): 145-152
- Sheridan, D., Carr, M., and Bucicarelli, L. (2010) “In at the deep end – open design for first year engineering students”, *Proc. of Science and Mathematics Education Conference (SMEC 2010)*, Dublin, Ireland.
- Verner, I.M., Aroshas, S., and Berman, A. (2008) Integrating supplementary application-based tutorials in the multivariable calculus course, *Int. J. Math. Educ. Sci. Technol.* 39(4): 427–442.
- Yates, P. (2003) Teaching mathematics to first year undergraduate chemists in the context of their discipline. *Maths for Engineering and Science LTSN MathsTEAM booklet*.
- Young, C.Y., Georgiopoulos, M., Hagen, S.C., Geiger, C.L., Dagley-Falls, M.A., Islas, A.L., Ramsey, P.J., Lancey, P.M., Straney, R.A., Forde, D.S., Bradbury, E.E. (2011) Improving student learning in calculus through applications. *International Journal of Mathematical Education in Science and Technology*. 42(5): 591–604.